From Strong Interactions to String Theory

Igor Klebanov

Princeton University

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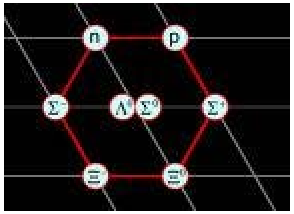
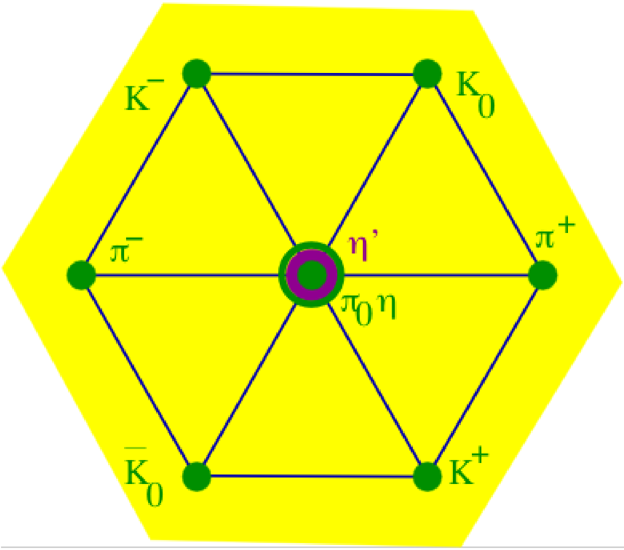
Introduction

- One often hears of string theory as the leading hope for unifying all known interactions-- strong (nuclear), electromagnetic, weak (β-decay) and gravitational-- into a consistent quantum theory. Some have dubbed it `The Theory of Everything.'
- Actually, string theory has had more humble beginnings. It was invented in the late 60's to model `just' the strong (nuclear) interactions.

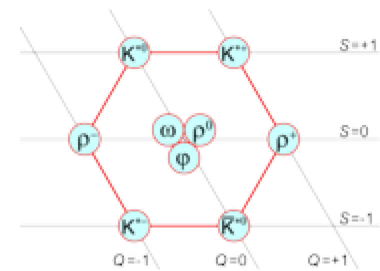
- The strong interactions are short range (~ 1 fm = 10 $^{-15}$ m), but much stronger than the electromagnetic.
- The strong interaction analogue $\alpha_s = g_{YM}^2/(4\pi)$ of the electromagnetic `fine structure constant' (α=1/137) is about 100 times bigger.
- As more and more hadrons were discovered, they were grouped into multiplets. For example, the lightest spin-0 mesons form an `octet.' This gave impetus to the Quark Model.

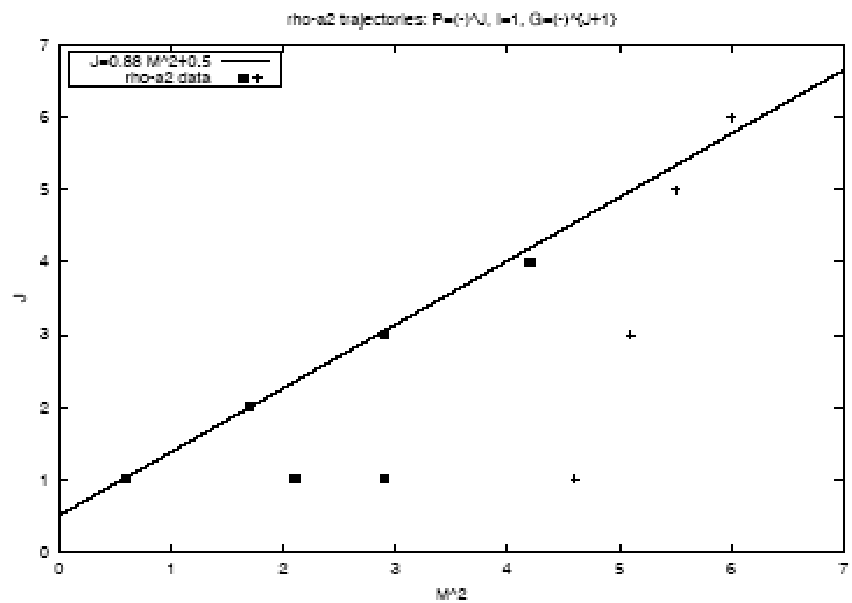
Gell-Mann; Zweig



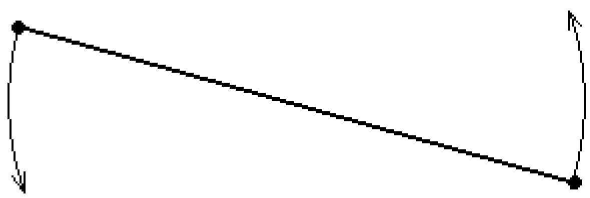


- Various heavier mesons, with higher intrinsic spin, have also been discovered.
- Early empirical evidence for the string-like structure of hadrons comes from arranging mesons and baryons into `Regge trajectories' on plots of angular momentum vs. mass-squared.
- A leading `Regge trajectory' of mesons is shown (ρ, a_2 …)





Open String Picture of Mesons

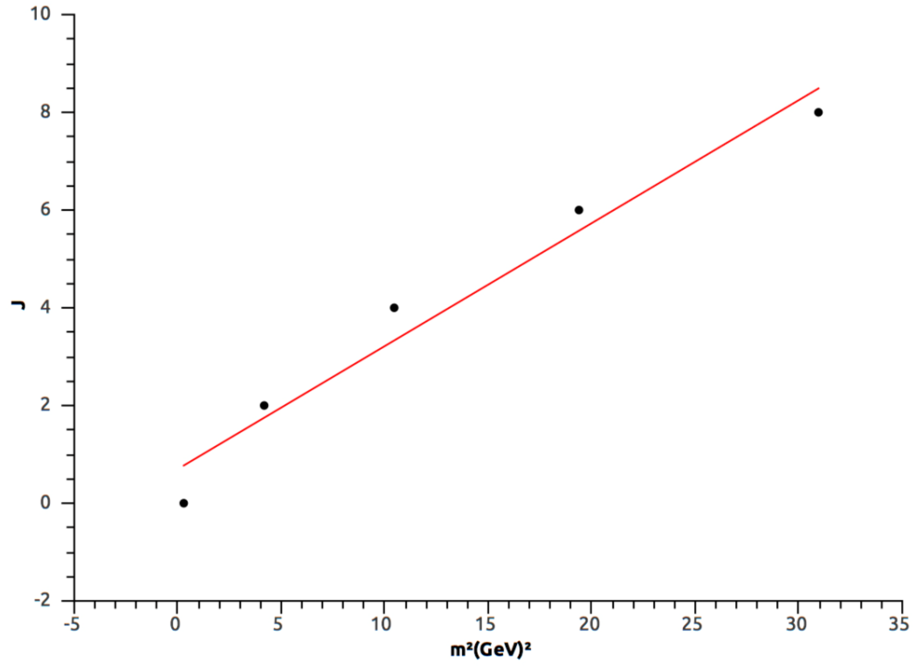


$$J = \alpha' m^2 + \alpha(0)$$

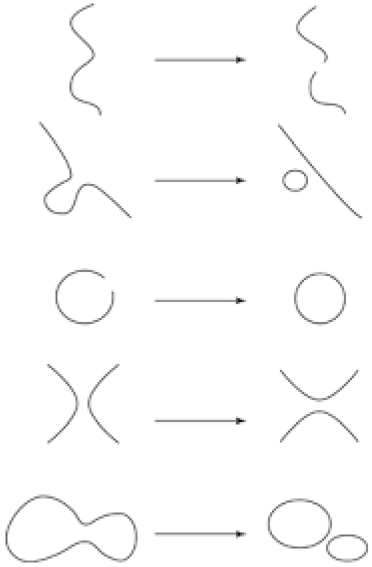
- In the string model, excited mesons are identified with excitations (rotational and vibrational) of a relativistic string of energy density ~ 1 GeV/fm, which is around 1.6 kiloJoules/cm.
- The rest energy of the spin-1 ρ meson is 0.78 GeV.
- The linear relation between angular momentum and mass-squared is provided by a spinning relativistic string. Later it was understood that a quark and anti-quark are located at the string endpoints.

Pomeron trajectories

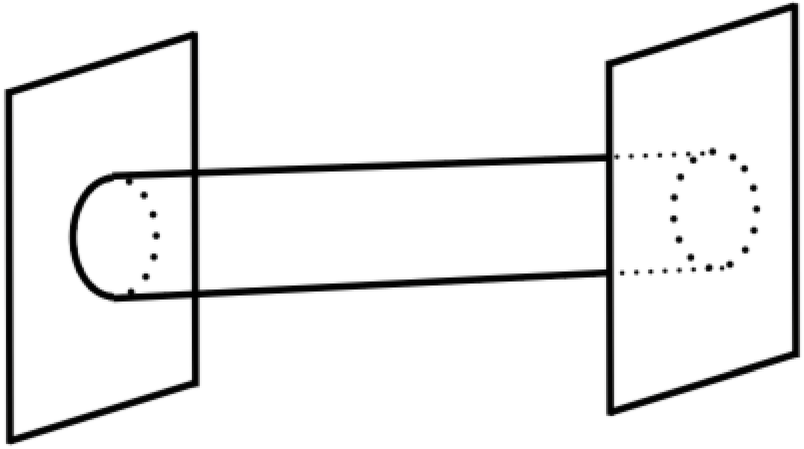
- A milestone in the physics of 1960s was the discovery of the additional trajectories, named in honor of Pomeranchuk, which make leading high-energy contributions.



- Possible string interactions:



- 2->2 scattering of open strings (mesons) via a closed string (pomeron) exchange.

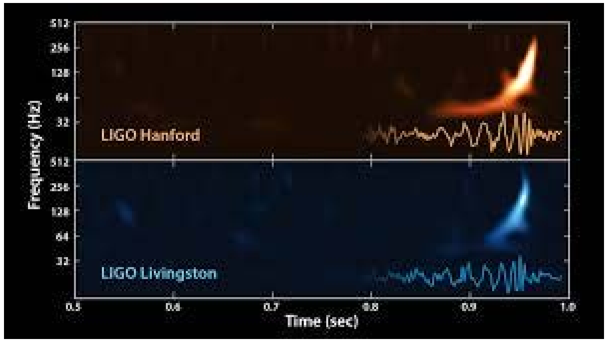


Supersymmetry

- String models in 4 spacetime dimensions ran into problems.
- Consistent string theories were discovered in 10 spacetime dimensions. Such theories have a remarkable new symmetry, called Supersymmetry (SUSY), which pairs up bosonic particles (integer spin) with fermionic particles (half-odd-integer spin). Such pairs are called superpartners.
- The relation of 10-dimensional superstring theories to strong interaction was completely obscure in the 70’s.

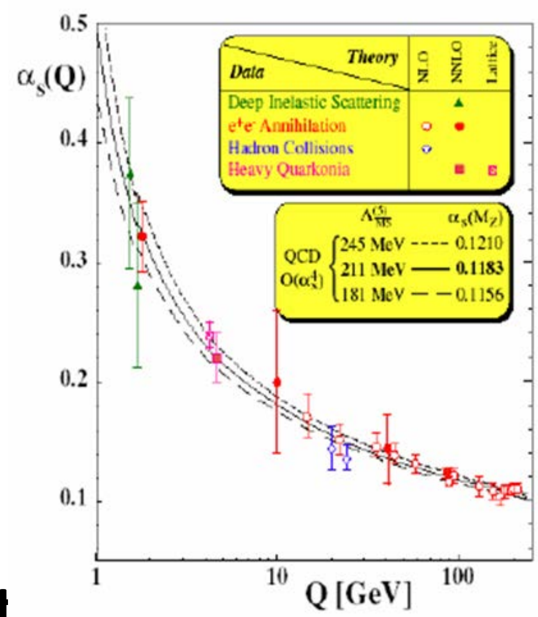
Towards Quantum Gravity

- For a number of reasons, most physicists gave up on strings as a description of strong interactions. Instead, string theory emerged as the leading hope for unifying quantum gravity with other forces. Scherk, Schwarz; Yoneya
- The massless spin-2 particle, graviton, predicted by Einstein's General Relativity, is the lightest vibrational mode of the closed superstring.
- Gravity waves were observed directly in 2015.

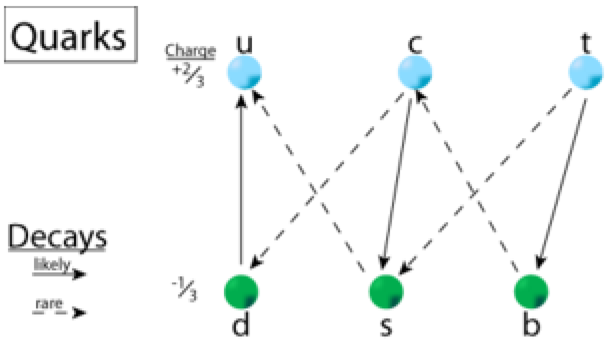


Quantum Chromodynamics

- In 1973 a point particle theory of strong interactions, inspired by the quark model, was proposed: the Quantum Chromodynamics – a Yang-Mills theory with gauge group SU(3).
- It exhibits asymptotic freedom: the interactions weaken at short distances.



- The hadrons are made of spin 1/2 constituents called quarks and spin 1 ones called gluons. Quarks come in 6 known flavors, and each flavor comes in 3 different color states.



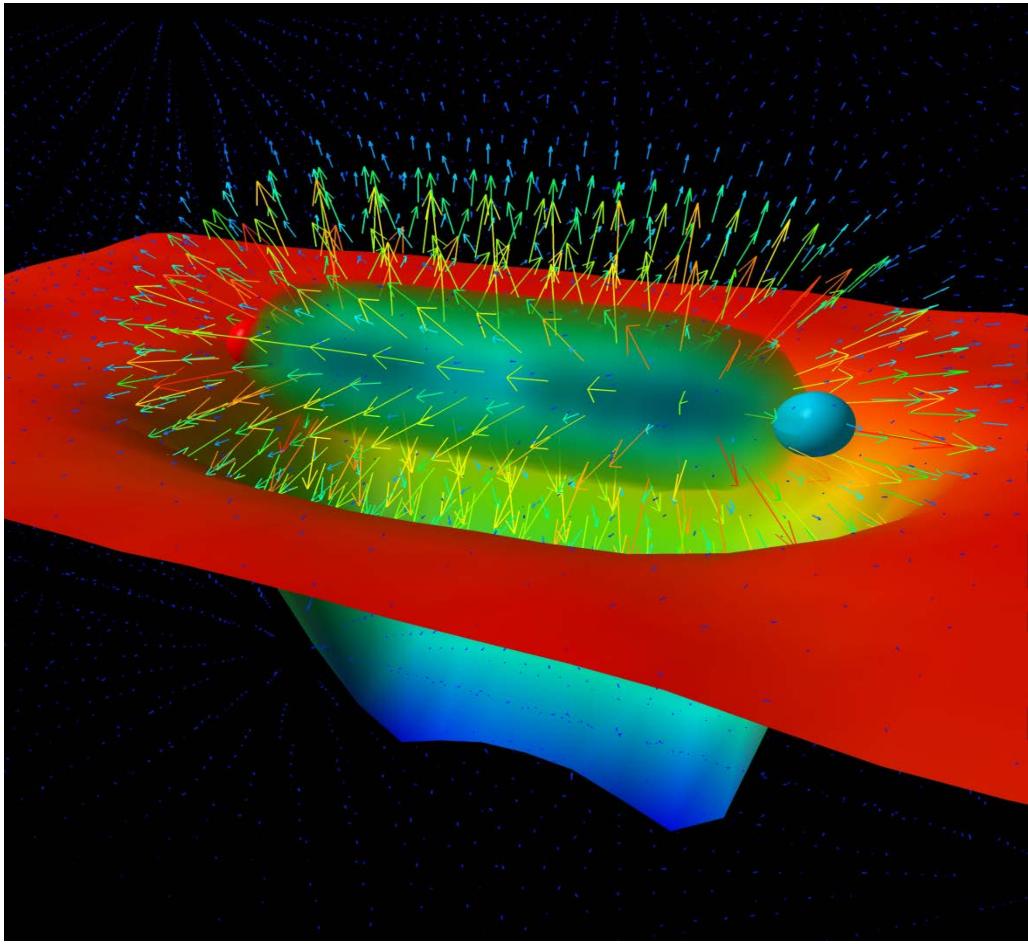
- The adjoint gluons come in eight different color states:

$$S = -\int d^4x \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu}^2$$

- The Pomeron trajectories are related to the existence of gluonic excitations.

QCD Gives Strings A Chance

- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is approximately described by the Nambu-Goto area action. So, strings have been observed, at least in numerical simulations of Yang-Mills theory.



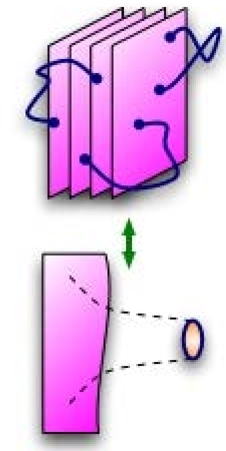
Large N Yang-Mills Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling fixed:

$$\lambda = g_{\mathrm{YM}}^2 N$$

- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.

D-Branes vs. Geometry



- Dirichlet branes led string theory back to gauge theory in the mid-90's. Polchinski
- A stack of N Dirichlet 3-branes realizes $\mathcal{N}$=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

whose radius is related to the coupling by $L^4 = g_{\rm YM}^2 N \alpha'^2$

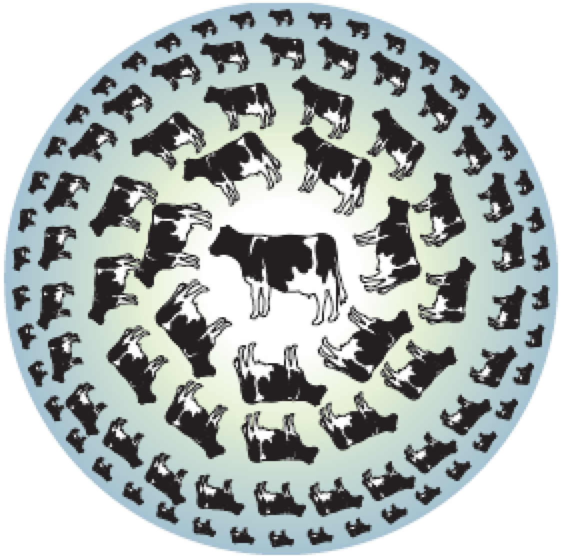
(Super) Conformal Invariance

- In the $\mathcal{N}$=4 Supersymmetric Yang-Mills theory the Asymptotic Freedom is canceled by the extra fields; the gauge coupling is independent of energy. The theory is invariant under scale transformations $x^\mu \to a\, x^\mu$.
- It is also invariant under space-time inversions.
- The conformal group in d+1 space-time dimensions is SO(2,d+1).
- It is widely hoped that this theory is the Harmonic Oscillator of 4-dimensional Quantum Field Theory.

The AdS/CFT Duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the $\mathcal{N}$=4 SYM theory this compact space is a 5-d sphere.
- The geometrical symmetry of the AdS_5 space realizes the conformal symmetry of the gauge theory.
- The AdS space-time is a generalized hyperboloid. It has negative curvature.



- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $\frac{L^2}{\alpha'} \sim \sqrt{g_{\rm YM}^2 N}$

- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of

$$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$

- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Their scaling dimensions are an important set of quantities

$$\langle \mathcal{O}_{\Delta_1}(x_1) \mathcal{O}_{\Delta_2}(x_2) \rangle = \frac{\delta_{\Delta_1, \Delta_2}}{|x_1 - x_2|^{2\Delta_1}}$$

- The operator dimension is related to mass of the corresponding field in AdS space:

$$\Delta_{\pm} = 2 \pm \sqrt{4 + m^2 L^2}$$

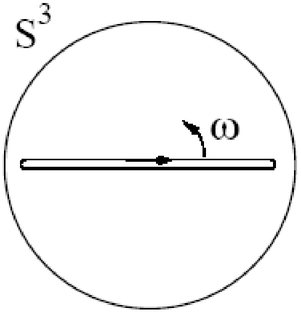
- The energy-momentum tensor corresponds to the graviton in AdS_5

Higher-Spin Operators and Spinning Strings

- The dual of a high-spin operator of S>>1

$$\mathrm{Tr}\, F_{+\mu} D_{+}^{S-2} F_{+}{}^{\mu}$$

is a folded string spinning around the center of AdS_5. Gubser, IK, Polyakov



- The structure of dimensions of high-spin operators is

$$\Delta - S = f(g) \ln S \; + O(S^0), \qquad g = \frac{\sqrt{g_{YM}^2 N}}{4\pi}$$

- Weak coupling expansion of the function f(g)

Kotikov, Lipatov, Onishchenko, Velizhanin; Bern, Dixon, Smirnov; …

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 + O(g^8)$$

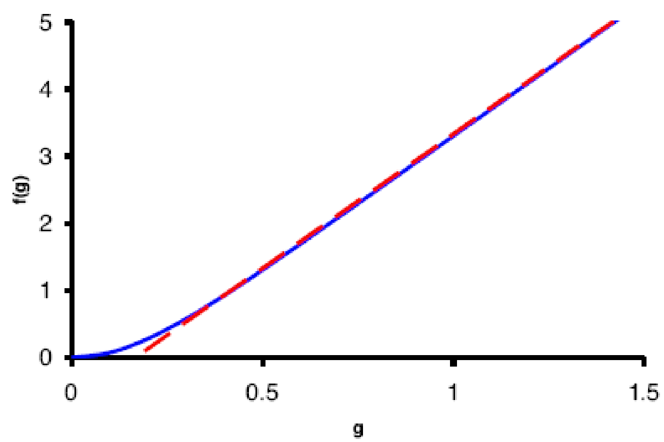
- At strong coupling, the AdS/CFT correspondence predicts via the spinning string energy calculation
- Gubser, IK, Polyakov; Frolov, Tseytlin

$$f(g) = 4g - \frac{3\ln 2}{\pi} + \dots$$

- Methods of exact integrability allow to match them smoothly.

Beisert, Eden, Staudacher;

Benna, Benvenuti, IK, Scardicchio



Entanglement Entropy

- Divide d-dimensional space into two complementary regions, A and B. Their quantum entanglement entropy is the entropy seen by an observer in A who does not have access to the degrees of freedom in B:

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

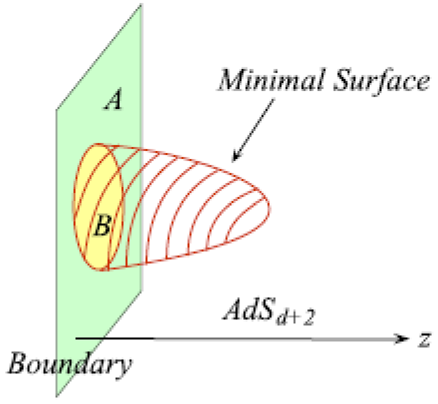
The reduced density matrix is

$$\rho_A = \mathrm{Tr}_B \rho_0 \qquad \rho_0 = |0\rangle\langle 0|$$

- In a QFT, the entanglement entropy is UV divergent and proportional to the volume of the boundary

$$S_A \simeq \frac{V_{d-1}}{a^{d-1}}$$

- In a d+2 dimensional gravity dual, the entanglement entropy is the area of the minimal d dimensional manifold γ which at the AdS boundary approaches the boundary between A and B
Ryu, Takayanagi



$$S_A = \frac{1}{4G_N^{(d+2)}} \int_\gamma d^d\sigma \sqrt{G_{\rm ind}^{(d)}}$$

- For d=1 this gives the expected result $S_A = \frac{c}{3} \log \frac{l}{a}$
Holzhey, Larsen, Wilczek; Cardy, Calabrese

- For d>1 we get predictions about EE of strongly coupled field theories.

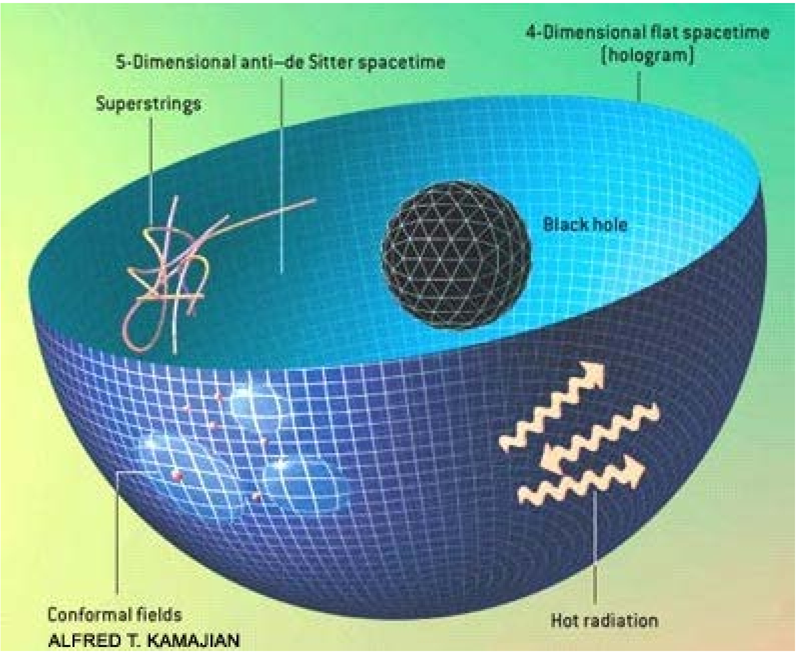
Thermal gauge theory

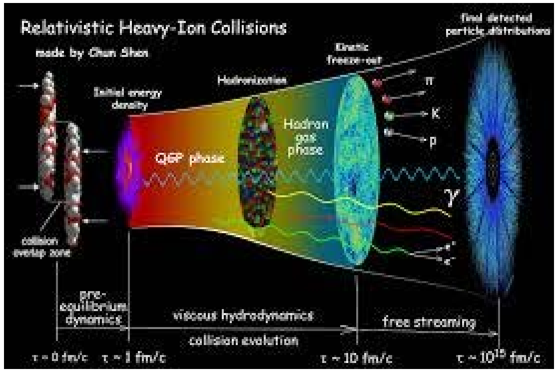
- Is described by a black hole at the center of AdS_5
- The event horizon contains Bekenstein-Hawking entropy

$$S_{BH} = \frac{2\pi A_h}{\kappa^2}$$

- A brief calculation gives the entropy density $s = \frac{\pi^2}{2} N^2 T^3$

Gubser, IK, Peet





Shear Viscosity η of the Plasma

- In a comoving frame, $$T_{ij} = \delta_{ij} p - \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k\right)$$
- Can be also determined through the

  Kubo formula

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\mathbf{x}\, e^{i\omega t} \langle [T_{xy}(t,\mathbf{x}), T_{xy}(0,0)] \rangle$$

- For the $\mathcal{N}$=4 supersymmetric YM theory this 2-point function may be computed from graviton absorption by the 3-brane metric.
- At very strong coupling, Policastro, Son and Starinets found

$$\eta = \frac{\pi}{8} N^2 T^3 \qquad\qquad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

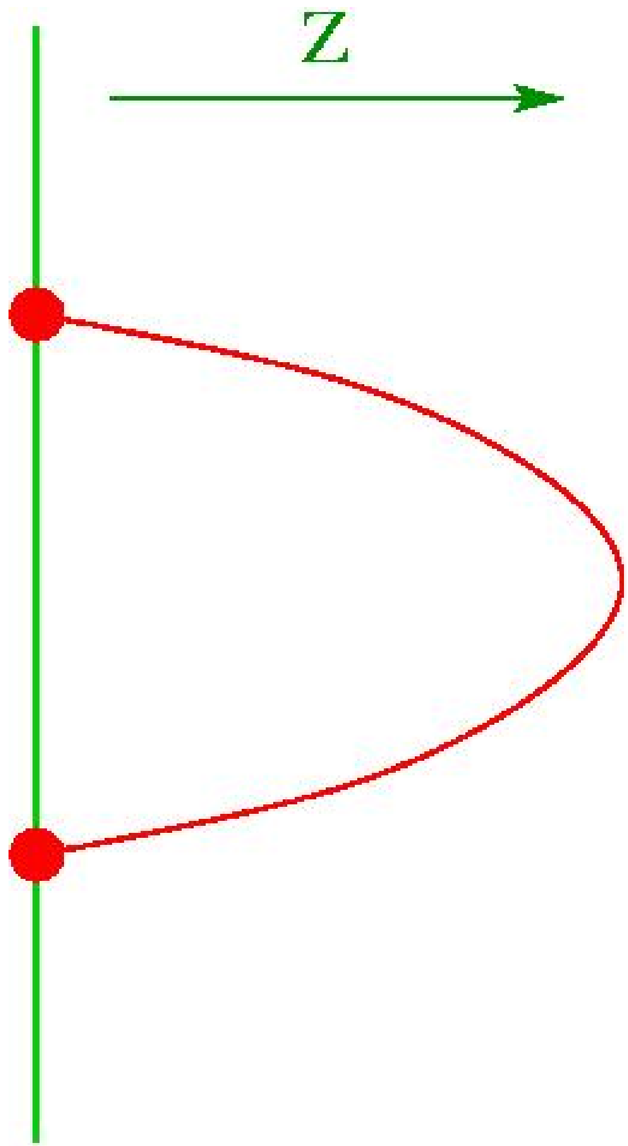
- Heavy ion collisions at RHIC and LHC produced a plasma with a comparably low value of η/s

The quark anti-quark potential

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



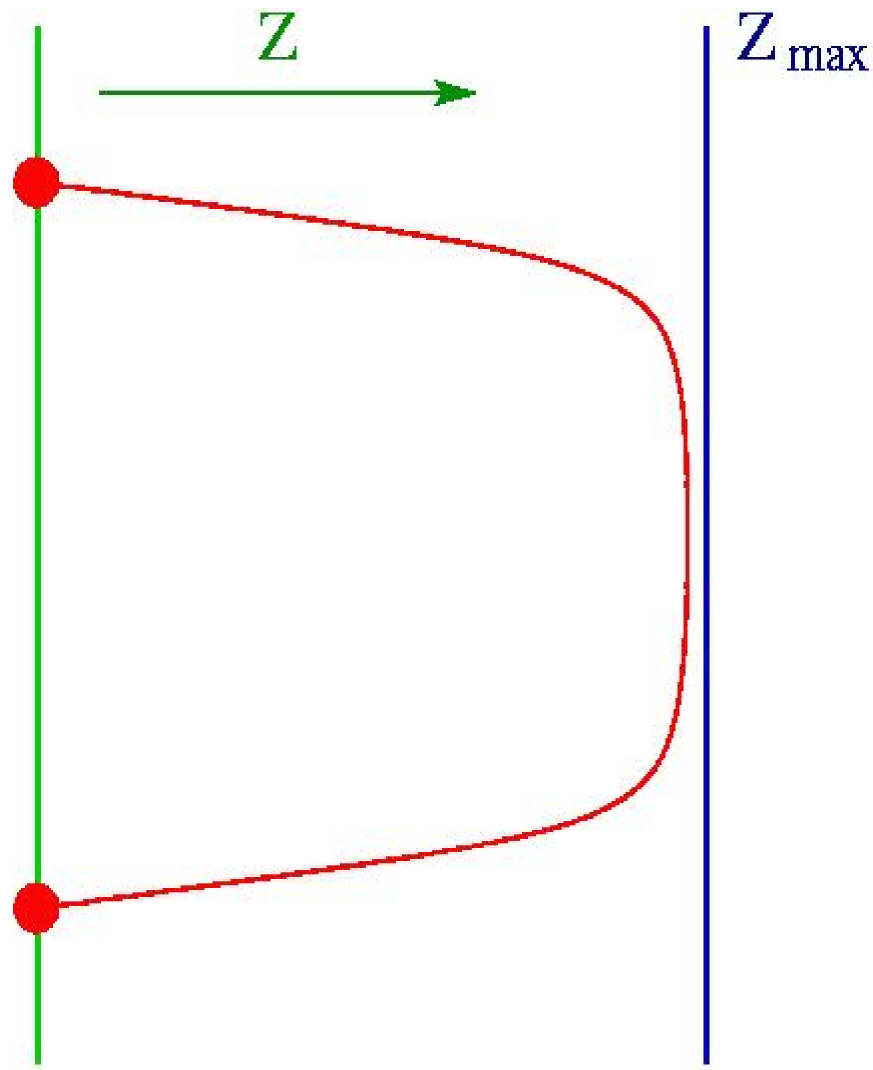
Color Confinement

- The quark anti-quark potential is linear at large distances but nearly Coulombic at small distances.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

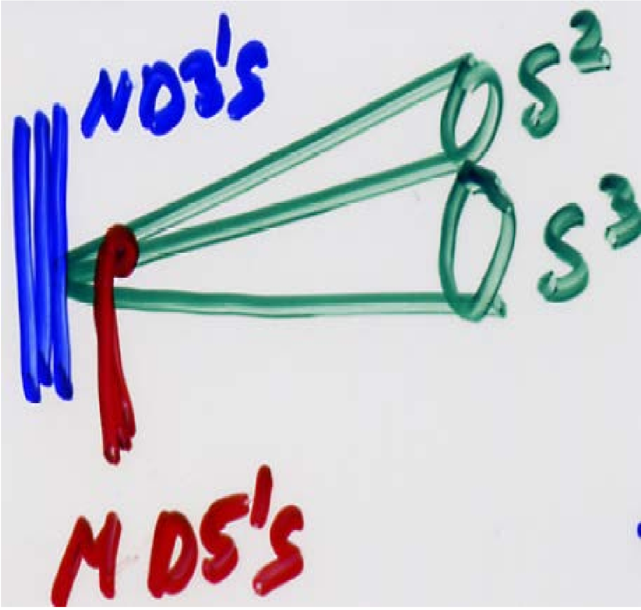
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, change the gauge theory: add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

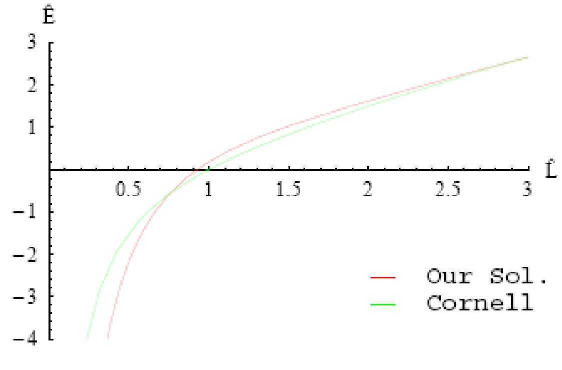
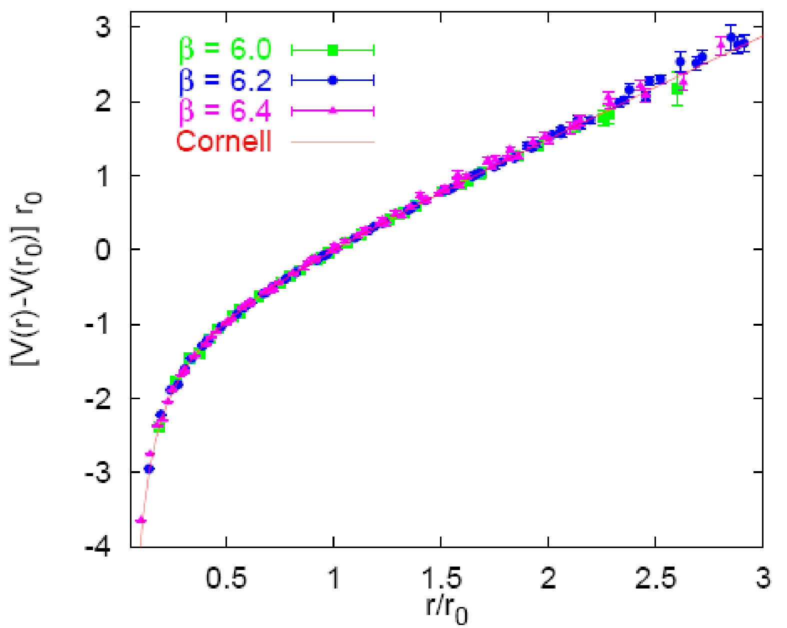


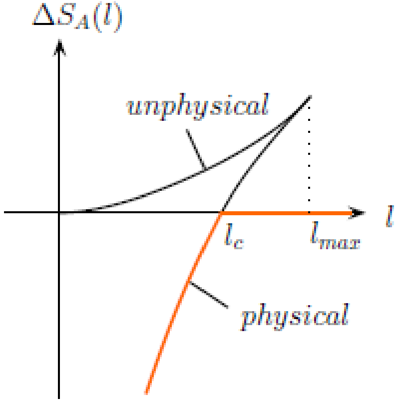
Figure 11: Comparison to the Cornell model



- Normal modes of the warped throat correspond to glueball-like bound states in the gauge theory.
- Their spectra have been calculated using standard methods of (super)gravity.

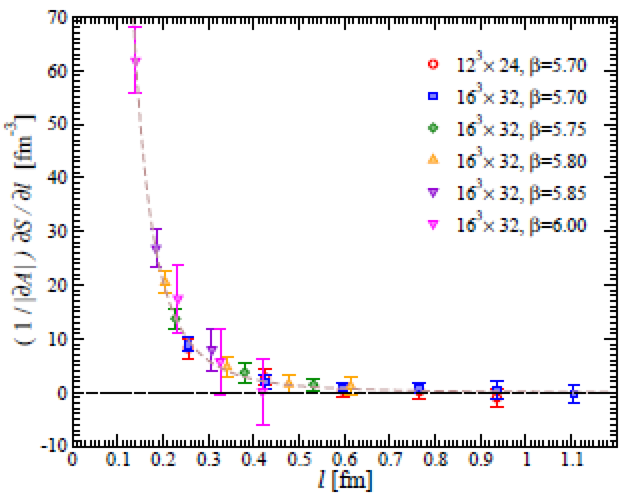
Confinement and Entanglement

- Due to the confinement, there is a phase transition in the behavior of the entanglement entropy as a function of the strip width.
  IK, Kutasov, Murugan; Nishioka, Takayanagi



- There is evidence of a similar transition or crossover in lattice gauge theory. Velitsky; Buividovich, Polikarpov;

  Nakagawa, Nakamura, Motoki, Zakharov



Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions.
- The Anti-de Sitter/Conformal Field Theory correspondence makes this connection precise. It makes many dynamical statements about strongly coupled conformal gauge theories, including the scaling dimensions of composite operators and quantum entanglement entropy.
- Allows for calculation of transport coefficients in strongly coupled gluonic plasma. Provides a model for what was observed at heavy ion colliders.
- Extensions of AdS/CFT provide a new geometrical understanding of color confinement and other strong coupling phenomena.