PRINCETON UNIVERSITY PHYSICS COMPETITION

# GENERAL INFORMATION

Online Part

## Guidelines

Student teams will have a total of **one week** to complete the exam from start to finish. We recommend that teams set aside approximately 20 hours to allow enough time for successful completion. All teams are required to submit their response with a cover page listing the title of their work, the date, and the information provided during registration. Additionally, it should include the signatures of all contestants on that team. Each submitted page should also have on it the team ID number and problem number. All other formatting decisions are delegated to the teams themselves, with no one style favored over another. While points will not be deducted for written work, we suggest that teams use a typesetting language (e.g., LaTeX) or a word-processing program (e.g., Microsoft Word/Pages) for convenience. There will be three types of problems on the online exam, with about twenty questions in total:

- **Introductory:** These questions will introduce students to the research topic. No prior knowledge is assumed.
- **Intermediate:** Questions that will bring students up to date with current physics research that assume knowledge of the introductory material and questions.
- **Research:** As the most difficult questions on the exam, research questions will test students' knowledge and creativity in manipulating and interpreting data from current research.

## Collaboration Policy

Students participating in the competition may only correspond with other members of their team. No other correspondence is allowed, including: mentors, teachers, professors, and other students. While teams are allowed to use a plethora of online resources, participating students are barred from posting content or asking questions related to the exam. As repeated below, teams are also welcome to utilize the Piazza page at `http://piazza.com/princeton_university_physics_competition/spring2015/pupc` and ask questions in case something is unclear in the assignment.

## Resources

Barring violating the collaboration policy, students have access to the following types of resources:

- **Online**: Teams may use any information they find useful on the Internet. However, under no circumstances may they engage in active interactions such as posting content or asking questions regarding the exam.
- **Piazza page**: Teams are encouraged to create an account in Piazza and register in the class at the following URL:

  `http://piazza.com/princeton_university_physics_competition/spring2015/pupc`

  The access code is: **firstedition**

  This way, you will be able to ask questions if you'd like to clarify something.

- **Published Materials**: Teams may take advantage of any published material (print/online)
- **Computational**: Teams may use any computational resources they might find helpful, such as Wolfram Alpha/Mathematica, Matlab, Excel, or lower level programming languages (C++, Java, Python, etc).

## Citations

All student submissions with outside material must include numbered citations. We do not prefer any style of citation in particular. Students may find the following guide useful in learning when to cite sourced material:

`http://www.princeton.edu/pr/pub/integrity/pages/cite/`

## Submission

Teams must submit their Online Part solutions by e-mailing **pupc.submit@gmail.com** in accordance with the Test Rules before 2 PM Eastern Time (UTC-5) on Saturday, November 22, 2014. Teams will not be able to submit their solutions to the Online Part at any later time. Regardless of internal formatting, solutions should be submitted as a single PDF document with the ".pdf" extension. The e-mail must contain your team ID in the Subject field. Only one person per team, identified as the "team manager" during registration, should send this e-mail. (Team managers will receive their team IDs via e-mail after the Online Part is released, by Sunday, November 16.) Each submitted page should also have on it the team ID number and problem number, and the front page should include the following handwritten pledge ("I pledge my Honor that this assignment represents my own work in accordance with regulations set forth herein") followed by the signatures of all contestants in that team. Any discrepancies will be dealt with by the current Director of PUPC.

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PRINCETON UNIVERSITY PHYSICS COMPETITION

# PUPC 2014: Condensed Matter Online Part

*If you choose to submit your solutions for the Condensed Matter part, you are NOT required to submit the Cosmology part!*

This part will contain background material in crystal structure, quantum theory, resistivity and conductivity, superconductivity, the process underpinning the growing field of topological insulators, and, finally, Kondo insulators, a subtopic that is at the forefront of groundbreaking research in present day condensed matter physics.

Each section of the background material is followed by several conceptual and/or applied questions that you are expected to answer in any order you choose (but the material is structured in a way that would lead you through the assignment sequence logically).

**Good luck!**

Condensed matter physics deals with the study of systems made up of condensed phases of matter, and a great deal of research focuses on analyzing such systems on a micro-level. The behavior of these systems is explored using theories from solid state physics, quantum mechanics, electrodynamics, and statistical mechanics. In this part, you will explore some important concepts one encounters in condensed matter.

## 1 Crystal Structure background

**Crystal lattices** Condensed matter is largely concerned with the study of crystalline materials, or those with a regular, repeating structure. One reason is that crystals are easier to study than irregular matter. More importantly, most real solid materials are crystalline, including quartz, salt, and metals.

The structure of a crystal can be described in terms of a crystal lattice (or Bravais lattice), which specifies the periodic geometry in which the crystal units (e.g. atoms) are arranged. Two equivalent definitions of a Bravais lattice are:

1. A Bravais lattice is an infinite array of discrete points with an arrangement and orientation that appears exactly the same, from whichever of the points the array is viewed
2. A (three-dimensional) Bravais lattice consists of all points with position vectors $\boldsymbol{R}$ of the form:

$$\boldsymbol{R} = n_1\boldsymbol{a}_1 + n_2\boldsymbol{a}_2 + n_3\boldsymbol{a}_3, \tag{1}$$

   where the $n_i$ are integers.

The vectors $\boldsymbol{a}_i$, or primitive vectors, are said to span the lattice. Any point in the lattice can be reached from any other point by a sum of primitive vectors. It should be noted that the choice of primitive vectors is not unique; in fact, there are an infinite number of equivalent choices. A picture of a Bravais lattice with one choice of primitive vectors is provided in Figure 1.

A Bravais lattice is infinite by definition. Although real crystals are of course finite, they contain a very large number of points. The points in the center are so far from the surface that they are generally unaffected by the surface, so they behave as if in an infinite solid.

One important Bravais lattice, the body-centerd cubic (BCC) lattice, is shown in Figure 2. It can be formed by starting with a simple cubic lattice (whose points we call A) and adding a point in the center of

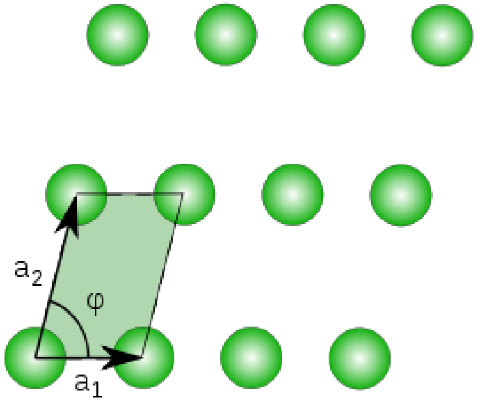


Figure 1: A 2D Bravais lattice with one set of possible primitive vectors shown.

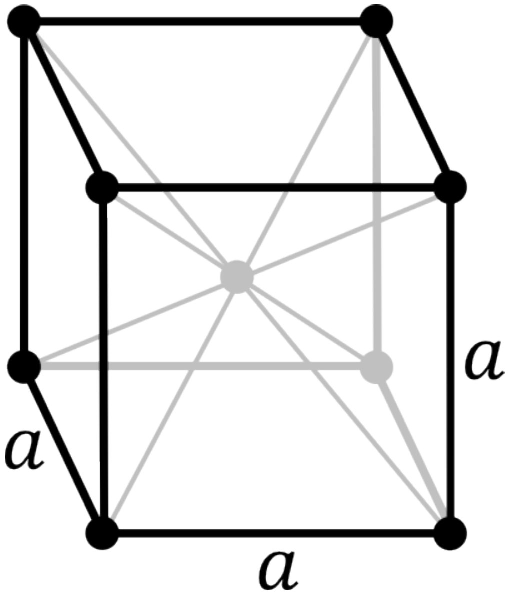


Figure 2: A body-centered cubic (BCC) lattice.

each cube (which we call B). The B points may appear to be fundamentally different from the A points, but they are not. To see this, note that the B points can be connected to form cubes, where now the A points lie in the centers. Thus, each point has identical surroundings, satisfying definition 1 of a Bravais lattice.

This implies that the BCC lattice can be described by a set of primitive vectors. One choice is:

$$\boldsymbol{a}_1 = a\hat{\boldsymbol{x}}, \quad \boldsymbol{a}_2 = a\hat{\boldsymbol{y}}, \quad \boldsymbol{a}_3 = \frac{a}{2}(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}) \tag{2}$$

A more symmetric choice is:

$$\boldsymbol{a}_1 = \frac{a}{2}(\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}} - \hat{\boldsymbol{x}}), \quad \boldsymbol{a}_2 = \frac{a}{2}(\hat{\boldsymbol{z}} + \hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}), \quad \boldsymbol{a}_3 = \frac{a}{2}(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}) \tag{3}$$

In Problem 1.1, you will show that these choices are equivalent.

## 2 Reciprocal Lattices

The reciprocal lattice is a fundamental concept in condensed matter physics. It is useful for analyzing many phenomena involving functions in a periodic lattice, including crystal diffraction and electronic wavefunctions in a crystal.

Consider a Bravais lattice with lattice vectors $\boldsymbol{R}$. Also consider a plane wave, or a wave propagating in one direction, which can in general be represented by the formula $e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$. (If this notation is not familiar, you will need to look up complex exponentials). A plane wave is defined by the wavevector $\boldsymbol{k}$, which points in the direction of wave propagation and has magnitude $\frac{2\pi}{\lambda}$, where $\lambda$ is the wavelength.

We say that a specific wavevector $\boldsymbol{k}$ has the periodicity of the lattice if, for every lattice vector $\boldsymbol{R}$, the wave is identical at two points separated by $\boldsymbol{R}$. Such a wavevector is a reciprocal lattice vector, and we give it the symbol $\boldsymbol{K}$. Mathematically:

$$e^{i\boldsymbol{K}\cdot(\boldsymbol{r}+\boldsymbol{R})} = e^{i\boldsymbol{K}\cdot\boldsymbol{r}}. \tag{4}$$

Factoring out the $e^{i\boldsymbol{K}\cdot\boldsymbol{r}}$, we say that a wavevector is a reciprocal lattice vector $\boldsymbol{K}$ if and only if

$$e^{i\boldsymbol{K}\cdot\boldsymbol{R}} = 1 \tag{5}$$

for each $\boldsymbol{R}$ in the Bravais lattice.

It turns out that the reciprocal lattice is itself a Bravais lattice, and thus there are an infinite number of choices of primitive lattice vectors. One easy choice is the following, in which we construct the reciprocal lattice vectors from the primitive vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$. Then we set

$$\boldsymbol{b}_1 = 2\pi\frac{\boldsymbol{a}_2\times\boldsymbol{a}_3}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)} \tag{6}$$

$$\boldsymbol{b}_2 = 2\pi\frac{\boldsymbol{a}_3\times\boldsymbol{a}_1}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)} \tag{7}$$

$$\boldsymbol{b}_3 = 2\pi\frac{\boldsymbol{a}_1\times\boldsymbol{a}_2}{\boldsymbol{a}_1\cdot(\boldsymbol{a}_2\times\boldsymbol{a}_3)} \tag{8}$$

To show that these are primitive vectors for the reciprocal lattice, we must show that any wavevector of the form

$$\boldsymbol{K} = k_1\boldsymbol{b}_1 + k_2\boldsymbol{b}_2 + k_3\boldsymbol{b}_3, \tag{9}$$

where the $k_i$ are integers, is indeed a reciprocal lattice vector, i.e. it satisfies Equation 5. First, it can be shown that

$$\boldsymbol{b}_i\cdot\boldsymbol{a}_j = 2\pi\delta_{ij} \tag{10}$$

where $\delta_{ij}$, the Kronecker delta symbol, is simply

$$\delta_{ij} = 0, i \neq j \tag{11}$$

$$\delta_{ij} = 1, i = j \tag{12}$$

Then for any $\boldsymbol{R} = n_1\boldsymbol{a}_1 + n_2\boldsymbol{a}_2 + n_3\boldsymbol{a}_3$, we have

$$\boldsymbol{k}\cdot\boldsymbol{R} = 2\pi(k_1n_1 + k_2n_2 + k_3n_3) \tag{13}$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{R}} = e^{i2\pi(k_1n_1+k_2n_2+k_3n_3)} \tag{14}$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{R}} = 1 \tag{15}$$

The last step is true because any complex exponential $e^{ix} = 1$, if $x$ is $2\pi$ times an integer.

## 2.1 Problem: Primitive vectors in FCC lattice

Derive a set of primitive vectors for the face-centered cubic lattice, pictured in Figure 2. More symmetrical choices are preferred. Then compute a set of reciprocal lattice vectors for the lattice.

## 2.2 Problem: Density of lattice points

A "primitive cell" is a volume of space that, when translated through all of the primitive vectors in a lattice, fills all of space without overlapping or leaving empty space. The choice is not unique, but any primitive cell contains exactly 1 lattice point (which may be divided, for example, into part of a lattice point at each corner). Demonstrate that, in a given family of parallel lattice planes, the density per unit area in a plane is $d/v$, where $v$ is the volume of a primitive cell and $d$ is the spacing between neighboring planes in the family.

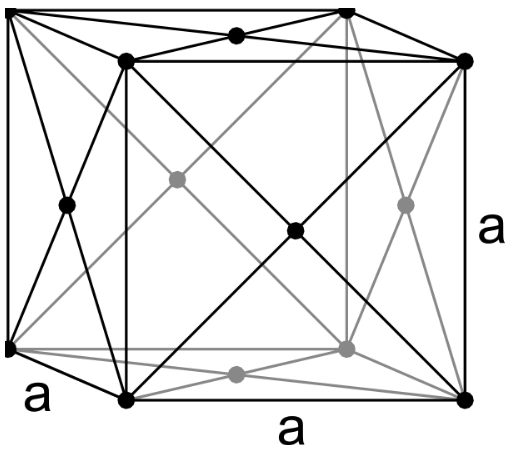


Figure 3: A face-centered cubic, or FCC lattice. This can be created from a simple cubic lattice by adding a point onto the center of each face, shared between two adjacent cubes.

### 2.3 Problem: BCC lattice

Show that any reciprocal lattice vector is perpendicular to each lattice plane in a given family of parallel planes. Using this fact, calculate the greatest density per area in a single lattice plane in a BCC lattice.

## 3 Quantum Mechanics background

**The Schrödinger Equation** Quantum mechanics is a fundamentally probabilistic way to understand physics at atomic and subatomic length scales. Much of classical intuition breaks down at this level. Richard Feynman quipped that quantum mechanics addresses “nature as She is -absurd”.

The fundamental equation of quantum mechanics (the $F = ma$ equivalent for quantum mechanics) is the Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

where $\hbar$ (pronounced “h-bar”) represents the reduced planck constant, $6.582eV * s$

Solving this equation gives $\Psi(x,t)$ , the wavefunctions. Wavefunctions are used to describe probabilitically where a particle is likely to be at a given time.

This is known as the Statistical Interpretation of the wavefunction, that is:

$$\int_a^b |\Psi(x,t)|^2 \, \mathrm{d}x = \text{probability of finding the particle between a and b, at time t}$$

**The Harmonic Oscillator** The classical paradigm of the harmonic oscilator is a mass $m$ attahced to a spring with spring contstant $k$. The motion is given by **Hooke’s law**:

$$F = m\frac{d^2x}{dt^2} = -kx$$

The solution to which is:

$$x(t) = Asin(\omega t) + Bcos(\omega t)$$

where

$$\omega = \sqrt{\frac{k}{m}}$$

is the angular frequency of the oscillation. The potential energy is parabolic:

$$V(x) = \frac{1}{2}kx^2$$

In fact, nearly any oscillatory motion is approximately simple harmonic and therefore has a parabolic potential, as long as the amplitude remains small.

In quantum mechanics, we want to solve the Schrödinger equation for the potential

$$V(x) = \frac{1}{2}m\omega^2x^2$$

for our purposes, the derivation itself is beyond the scope of this problem. After solving, we arrive at an equation for the ground state (that is, the lowest energy wavefunction which is a solution) wavefunction:

$$\Psi(x) = (\frac{m\omega}{\pi\hbar})^{1/4}e^{-(\frac{m\omega}{2\hbar}x^2)}$$

More importantly, the allowed energy of the groundstate is

$$E = \frac{1}{2}\hbar\omega$$

Because energy is quantized in quantum machincs, there are distinct allowed energies, each for it's corresponding allowed wavefunction. For the nth wavefunction, the corresponding energy is given by

$$E_n = (n + \frac{1}{2})\hbar\omega$$

### 3.1 Problem: Probability of Locating a Particle

What is the probability of finding a particle (which is in the ground state of the harmonic oscillator) outside of the classically allowed region? The classically allowed region goes from $-\sqrt{2E/m\omega^2}$ to $+\sqrt{2E/m\omega^2}$. Why is this region classically allowed? Why may the particle be found outside this region?

Note: some material in this section was derived from David Griffith's Introduction to Quantum Mechanics 2nd ed, Pearson (2006).

## 4 Energy Considerations

Now that you are familiarized with both lattice structure from Part 1 and the paradigm of the harmonic oscillator from Part 2, let's work with understanding energies in more depth.

### 4.1 Problem: Density of States

1. In virtually any condensed matter system the density of states (DOS) is very important. It represents how many states are at a given energy and is defined such that $D(E)\delta E$ is the number of states between energy levels $E$ and $E + \delta E$. In order to get a handle on this subtopic, please familiarize yourselves with this concept by referencing outside literature (we recommend textbook authors such as Kittel, Ashcroft and Mermin, Ibach and Luth, but it is described in various sources) and derive the density of states (as a function of energy) for:

1a. one-dimensional electron gas (line in $x$-direction),

1b. two-dimensional electron gas (plane in directions $x$ and $y$), and

1c. three-dimensional electron gas (volume in directions $x$, $y$, and $z$).

In each of the three cases, start with the single-particle energy expression and the wave vector. Hint: for the one-dimensional case the single-particle energy is $E_0 = \frac{(\hbar k_x)^2}{2m}$ and the wave vector is $k_x = \frac{2\pi}{L}n_x$, where $\frac{L}{2\pi}$ is the number of states per unit wave vector. In each subsequent dimension you will need to factor in momentum $k$ in all proper direction(s).

You may be surprised to find out that for each dimension the density of states is proportional to different powers of energy!

2. In each of the above three cases, please create a graph (either by hand, or, preferably, using your favorite computational software that you are familiar with, if any), plotting the density of states vs. energy in the energy range 0 eV to 10 eV. To do this you will need to look up numerical values of Planck constant $\hbar$ in $m^2kg/s$ and the mass of the electron (in kg). Please label your graphs. [Note: the plots do not have to be completely to scale; the most important part is to observe the behavior of the density of states for each of the three cases as a function of energy.]

### 4.2 Problem: Phenomenon on Lattice

Feel free to imagine yourselves as "micro-level" observers of the following interesting phenomenon. You encounter a two-dimensional lattice consisting of $N$ ions (of mass $M$) in a square array with lattice spacing $a$. At temperature T, the lattice energy is expressed as a sum over all phonon modes:

$$U(T) = 3\sum_{i=1}^{n}[n_k(T) + \frac{1}{2}]h\omega(k)$$

with $n_k(T) = \frac{1}{e^{h\omega(k)/kT}-1}$, which is the Bose-Einstein distribution. (A phonon describes the vibrational motion of a lattice, and you may wish to look up this concept in more depth in external sources.)

a. Using the Debye approximation $\omega(k) = c\,|k|$ (with $c$ being a constant), express the Debye wave vector $k_D$ in terms of $a$, $N$, $c$, and $\omega$. The two-dimensional density of states (re:previous problem) has the form

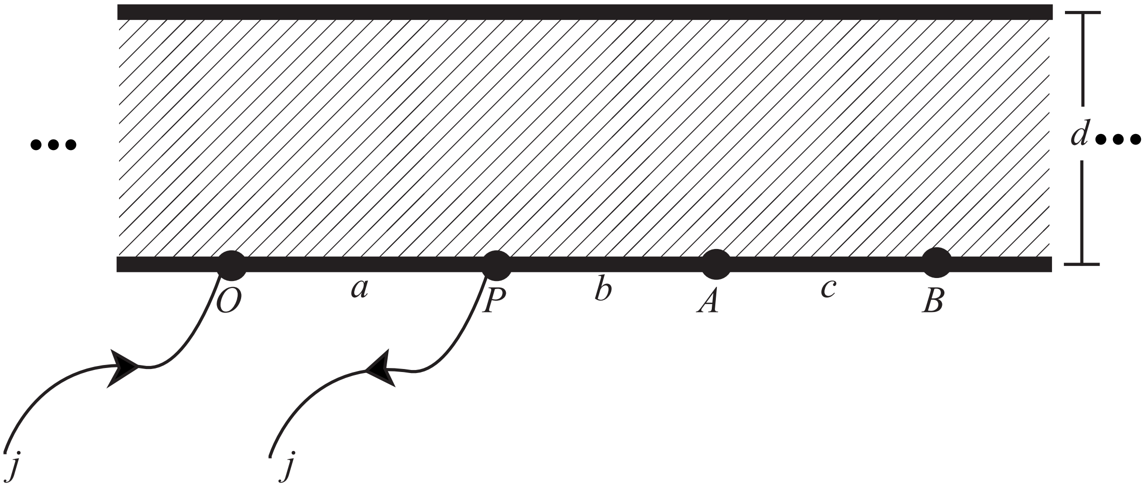
$$D(w) = \beta w$$

Express $\beta$ (hint: it is already encompassed in the expression for the Debye wave vector you just derived).

b. Starting from the above expression for U(T), please show that at high T (much greater than $\Omega_D$) the heat capacity approaches the Dulong-Petit value $3Nk_B$. You can ignore the constant zero-level energy term in U(T).

c. Show that at low T (much less than $\Omega_D$) the heat capacity behaves as $\alpha T^n$. Please derive $\alpha$ and $n$. Again, you can ignore the constant zero-level energy term in U(T).

# 5 Resistivity and Conductivity



1. Consider a thin infinite plate with the thickness $d$ and conductivity $\sigma$. At the point $O$, a current $j$ is injected. Determine the current density $J$ at radius $r$ on the surface of the plate!

   You can assume that the $d \ll r$.

2. We will also now pull out a current $j$ from point $P$ on the surface that is located as distance $a$ from point $O$. Determine the electric field at at point $A$ that is located at radius $r$ from point O on the surface!

3. Calculate the potential difference $V_{AB}$ between point $A$ and $B$ as described by the figure!

4. From this point we will create a new notation of material resistivity between two points of the sheet $R_{12}$ by measuring the voltage difference between two other points $V_{34}$ as,

   $$R_{12,34} = \frac{V_{34}}{I_{12}}$$

   Calculate the resistivity of all possible combination of $O$, $P$, $A$, and $B$, and determine that all of them satisfies the general relations of sheet resistivity namely,

   $$R_{12,34} = R_{34,12} \tag{16}$$

   $$e^{-\sigma\pi R_{12,34}d} + e^{-\sigma\pi R_{23,41}d} = 1 \tag{17}$$

   Both of the equation 16 and 17 will hold for any general shape of conductor with thickness $d$.

# 6 Band Gap Theory background

Before delving into the topics of superconductivity and Kondo insulators, it is necessary to briefly introduce the concept of band theory and ordinary insulators.

In semiconductors, the overlap between orbitals of ions located near one another generates narrow energy levels called bands. The valence band is the highest occupied band, the conduction band is the lowest unoccupied band, and these bands are separated by an energy gap called the band gap.

In real materials conduction occurs when there is either a presence of electrons in the conduction band or the presence of holes (absence of electrons) in the valence band. When an electron is excited from the valence band, it leaves a hole, and another electron in the valence band can fill this hole, leaving behind a hole of its own. An n-type semiconductor is produced when impurities inside the material ionize and release electrons into the conduction band, such that the net charge of the carriers is negative. A p-type semiconductor is produced when the impurities capture electrons and release holes into the valence band, such that the net charge of the carriers is positive.

In a conductor (metal) there is no energy gap as the two bands overlap. In insulators and semiconductors there must be an input of energy for electrons to make transitions between the bands as the lowest energy in the conduction band is higher than the highest energy in the valence band. Such energy input is impossible in insulators and the valence band is filled to capacity while the conduction band is empty; there will be a chemical potential between the two bands and the electron movement is prohibited. Please note that in this discussion the concept of band gap structure was applied to the bulk of the material, not its surface.

# 7 Superconductivity background

Superconductivity is a phenomenon that occurs below a certain critical temperature $T_C$ in certain materials, at which point the electrical resistance vanishes. It is usually described in terms of BCS theory, proposed in 1957, which postulates that superconductivity is caused by pairs of electrons called Cooper pairs condensing into a less energetic boson-like state, so that with a decrease of temperature more pairs form until all possible combinations are formed at the theoretical T = 0 K. The pairing thus contributes to an energy gap between single and paired-electron states. Superconductivity can be analyzed in terms of differential conductance across the materials interface, which is a typical transport measurement. Recently, a number of studies have been conducted with the goal of inducing superconductivity in TIs by combining it with a superconductor. One of the present-day directions in electrical transport research relies on the point-contact method similar to the one introduced in the above questions in the Resistivity and Conductivity section. For this section we suggest you familiarize yourselves further with two types of superconductors and the concept of Landau levels. [Note: this latter topic should not be hard because you were already introduced to the quantum harmonic oscillator in the Resistivity and Conductivity section above, and the energy of a system of non-interacting charged particles is found in the same way as that of a standard quantum harmonic oscillator.]

Based on what you learned, please answer the following conceptual questions.

### 7.1 Problems: Superconductivity

1. Explain what happens to the electrons under a certain critical temperature, and what kind of interaction is felt that is broken up at a higher temperature (and why).
2. How is a type II superconductor different from a type I superconductor?
3. In your own words, please explain briefly what a Landau level represents.
4. What happens to the energy gap between the Landau levels with increasing magnetic field?

## 8 Topological insulators and photoemission spectroscopy

Topological insulators (TIs), a class of materials that have been first experimentally observed in crystals only six years ago – here at Princeton, – act as electrical insulators in the bulk and as electrical conductors at the surface. While you are encouraged to familiarize yourselves with the theory (we recommend the paper entitled Colloquium: Topological Insulators by Hasan and Kane, 2010), we will not discuss them in depth in this assignment. The so-called angle-resolved photoemission spectroscopy, or ARPES, is needed to understand the properties of topological order in the three-dimensional (3D) TIs. This process consists of bombarding the 3D TI sample with photons of a predetermined momentum and energy. By means of the photoelectric effect, the sample absorbs the photons and emits electrons, which are then detected. The goal of this process is to relate energy and momentum of the emitted electrons to ultimately understand band dispersion of the sample.

### 8.1 Problem: Photoelectric effect

a. Please explain in your own words how the photoelectric effect works and calculate the maximum kinetic energy of an ejected electron from the surface of a TI if the work function is 4.5 eV (a typical value in recent experiments with laser-ARPES) and the frequency of the incident light ranges from 1 to 16 * $10^{14}$ Hz.

b. Please plot the values you found above and specify the condition for which the photoelectric effect will occur. After you have done this, you may find pleasure in knowing that this process underpins all real experiments performed at photoemission spectroscopy beam lines across the world at this precise moment!

## 9 Kondo Insulation background

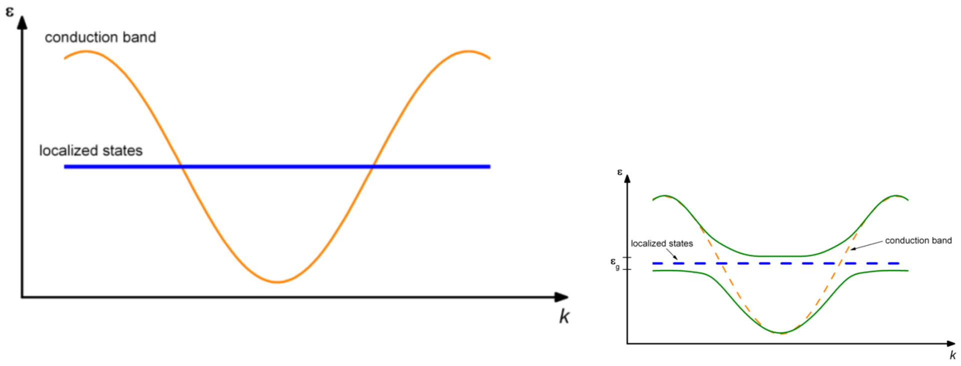
Kondo insulators are materials with strongly correlated electrons that open up a narrow band gap (approx. 10 meV) at low temperatures, and whose chemical potential (otherwise known as the Fermi level) lies in the gap. In contrast, in heavy fermions the chemical potential is located in the conduction band, as pictured in Fig. 1a.

As shown in Fig. 1b, the band gap opens up at low temperatures because of hybridization of localized electrons (in particular, f-electrons) with conduction electrons, which is called the Kondo effect. The Kondo effect describes the scattering of conduction electrons in a metal due to magnetic impurities, leading to the change in electrical resistivity with temperature.

Below is a typical temperature dependence of the resistivity that takes into account the Kondo effect:

$$\rho(T) = \rho_0 + aT^2 + c_m ln\frac{\mu}{T} + bT^5,$$

where $\rho_0$ is the residual resistance, $aT^2$ is the contribution from the Fermi liquid properties, and $bT^5$ corresponds to lattice vibrations or phonon scattering ($a$, $b$ and $c_m$ are constants).



(a) Chemical potential in the conduction band in heavy fermions.

(b) Localized (f) electrons hybridize with conduction electrons, opening up a band gap (at low temperatures).

Figure 4: Drawings corresponding to the Kondo Insulation background

More simply, the resistivity from the Kondo contribution and the electron-phonon scattering can be given by[3]:

$$\rho(T) = aT^5 - bc\ln(\frac{T}{T_F}),$$

where $aT^5$ corresponds to the phonon scattering.

Using this information, please answer the following questions.

## 9.1 Problem: Minimum Resistivity

1. Please derive the temperature at which the minimum resistivity from the Kondo contribution is observed (this should be a very quick calculation!).

Note: the expression you obtain above has been experimentally confirmed a number of times on alloys such as Fe in Au. (See Kondo, J. Prog. Theor. Phys. 1964.)

2. Please make a plot of what you think the resistivity vs. temperature curves would look like in a hypothetical alloy, including behavior below and above the temperature you found in the problem above.

3. You are presented with a set of known Mo-Nb alloys of various respective concentrations given in the graph below. From this plot, what can you conclude about the alloys with a concentration of 0.2 and 0.4 molybdenum, in contrast to the behavior of the other four alloys, particularly the ones with Mo concentration exceeding 0.7? Why do you think this is the case?

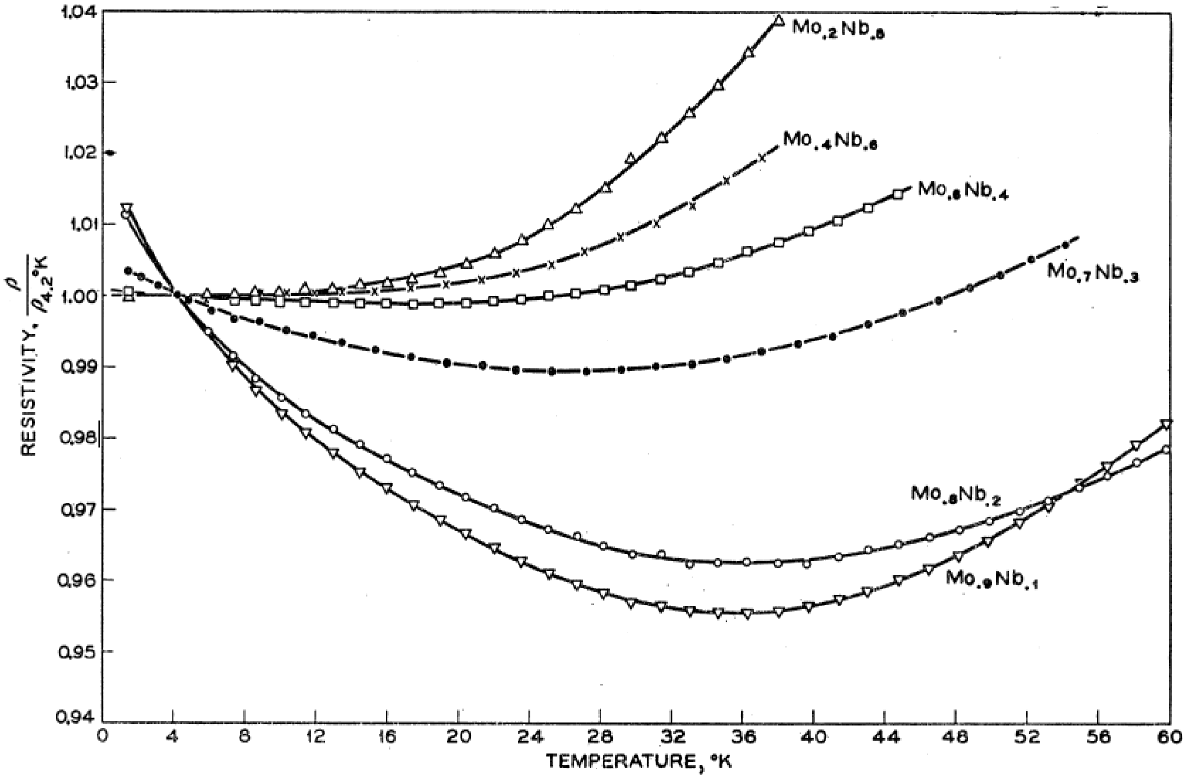


Figure 5: Graph pertaining to Problem 9.1.3.