Onsite Test 2015 Directions

The test consists of **four** questions which you will be given **2** hours to complete. **No collaboration** is allowed and partial credit will be given for incomplete solutions. Please place your name **both** on this packet and on your answer booklet. Only answers written in your answer booklet will be considered.

Some useful test-taking hints:

1. You may not be able to complete every problem. Keep moving; do what you know first.

2. Make your answer clear by circling it.

3. Use symbols rather than numbers wherever possible and check units.

4. Whenever possible, check whether an answer or intermediate result makes sense before moving on.

5. If you get stuck on an early part of a problem, check the later parts – they may be independent.

6. If you get stuck on an early part of a problem, and a later part depends on it, clearly define a symbol for the unknown answer and use it in later parts. However, keep in mind that we often give multiple parts to guide you through a problem.

To get full credit you need to show your work! Partial credit will also be awarded at the judges' discretion.
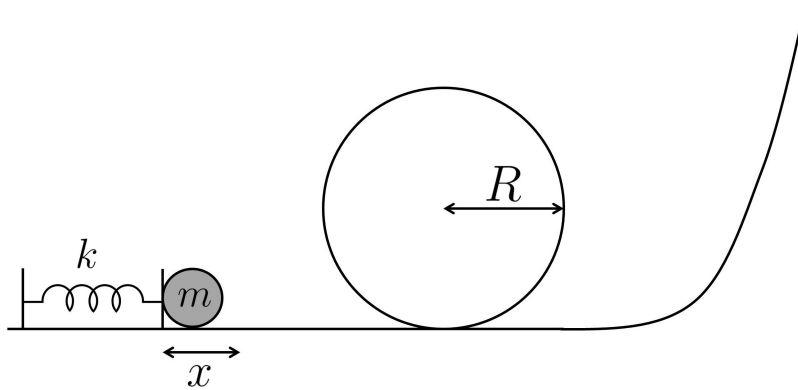
Good luck!

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Question 1.

A sphere with radius r and moment of inertia $I = \frac{2}{5}mr^2$ compresses a spring with spring constant k a distance x.

(a) How far must the spring be compressed so that the sphere rolls around the interior of a circular loop of radius R, depicted in the figure above, while always remaining in contact with the track?
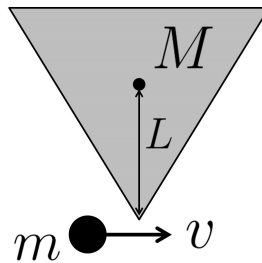
(10 points)

(b) In the case described in part (a) what height will the sphere reach at the end of the track?

(5 points)

Question 2.

A point particle with mass m and velocity v hits the edge of an equilateral triangle with mass M and moment of inertia $I = \frac{1}{4}ML^2$. The particle and the solid triangle are located on a flat frictionless surface.

(a) What is the velocity of the particle v'_m, velocity of the triangle v'_M, and angular velocity of the triangle ω'_M after the collision?

(15 points)

(b) Under certain conditions, a second collision will occur between the rotating triangle and the point particle. Determine the ratio m/M, so that a second collision occurs.
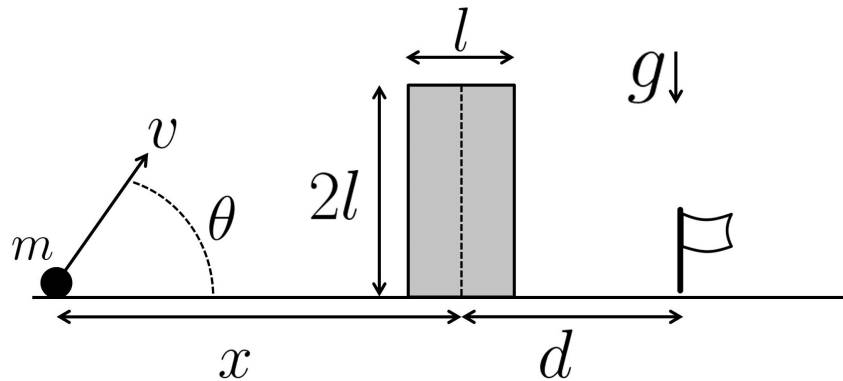
(5 points)

(c) After the second collision, what is the velocity of the particle v''_m, velocity of the triangle v''_M, and angular velocity of the triangle ω''_M ?

(10 points)

Question 3.

A point particle of mass m is launched over a wall of width l and height $2l$.

(a) Determine the distance from the wall x, the velocity v, and the angle θ from which the particle should be launched, so that it lands a distance d away from the center of the wall and the energy used $(\frac{1}{2}mv^2)$ is minimized.
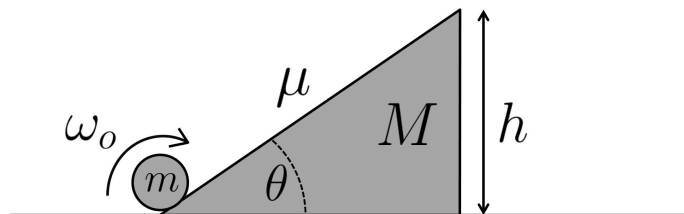
(10 points)

(b) If we just want to ensure that the ball goes over the wall (without any concern about where it lands) find x, v, and θ such that the energy used $(\frac{1}{2}mv^2)$ is minimized.

(15 points)

Question 4.

Consider an inclined plane of mass M which makes an angle θ with respect to the horizontal. The incline, initially at rest, is free to move on the frictionless horizontal. A solid sphere of mass m is spun up to an angular velocity ω_0 (initial linear velocity of zero) and released at the bottom of the incline. The ball experiences a kinetic frictional force on the incline with coefficient μ. Assume gravity points downward with magnitude g.

(a) Once the ball reaches the top of the ramp, what is the velocity v'_m of the ball, angular velocity ω'_m of the ball, and velocity v'_M of the ramp? Note: the ball rolls while slipping all the way to the top of the ramp.

(15 points)

(b) When the ball reaches the top, how much energy is lost?

(5 points)

(c) What is the minimum ω_0 such that the ball reaches the top of the ramp?

(10 points)